

Hong Kong Mathematics Olympiad (1998 – 99)

Heat Event (Individual)

香港数学竞赛 (1998 – 99)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

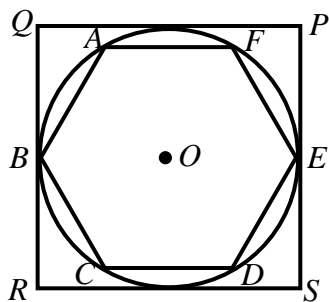
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 有一圆，其圆周是  $14\pi$  cm。若一弧所对的圆心角是  $\frac{1}{7}$  个弧度，设这弧的长度是  $X$  cm，求  $X$  的数值。

The circumference of a circle is  $14\pi$  cm. Let  $X$  cm be the length of an arc of the circle, which subtends an angle of  $\frac{1}{7}$  radian at the centre. Find the value of  $X$ .

2. 在图一， $ABCDEF$  是一正六边形及其面积是  $3\sqrt{3}$  cm<sup>2</sup>。设正方形  $PQRS$  的面积是  $X$  cm<sup>2</sup>，求  $X$  的数值。

In the figure,  $ABCDEF$  is a hexagon with area equal to  $3\sqrt{3}$  cm<sup>2</sup>. Let  $X$  cm<sup>2</sup> be the area of the square  $PQRS$ , find the value of  $X$ .



图一

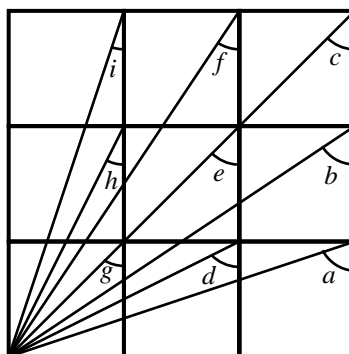
Figure 1

3. 已知 8 点，其中没有任何 3 点是共线的。求以任意 3 点作为三角形顶点的三角形的个数。

8 points are given and no three of them are collinear. Find the number of triangles formed by using any 3 of the given points as vertices.

4. 在图二，有一个  $3 \times 3$  正方形。设  $\angle a + \angle b + \dots + \angle i = X^\circ$ ，求  $X$  的数值。

In the figure, there is a  $3 \times 3$  square. Let  $\angle a + \angle b + \dots + \angle i = X^\circ$ , find the value of  $X$ .



图二

Figure 2

5. 在 0 至  $10^6$  之间, 有多少个整数  $n$ , 使得  $n^3$  的个位数字是 1?

How many integers  $n$  are there between 0 and  $10^6$ , such that the unit digit of  $n^3$  is 1?

6. 已知  $a$ 、 $b$ 、 $c$  是正整数, 且满足  $a < b < c = 100$ , 求以  $a$  cm、 $b$  cm、 $c$  cm 为边长的三角形的个数。

Given that  $a$ ,  $b$ ,  $c$  are positive integers and  $a < b < c = 100$ , find the number of triangles formed with sides equal  $a$  cm,  $b$  cm and  $c$  cm.

7. 一班青年参加旅行, 他们同意所有消费平均摊分。整个活动, 他们共用去 288 元。其中有一位成员无法支付其所应付出的部份。其它成员愿意各多付 4 元, 凑够其数。问共有多少青年参加这次旅行。

A group of youngsters went for a picnic. They agreed to share all expenses. The total amount used was \$ 288. One youngster had no money to pay his share, and each of the others had to pay \$ 4 more to cover the expenses. How many youngsters were there in the group?

8. 某两位数其值等于它的位值的和的 4 倍。若将该数的个位和十位数字相调，这个新两位数的值比其位值的和的 5 倍多出 18。求该数。

A two-digit number is equal to 4 times the sum of the digits, and the number formed by reversing the digits exceeds 5 times the sum of the digits by 18. What is the number?

9. 已知下列序列的第 1001 项的分母为 46，求该项的分子。

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$$

Given that the denominator of the 1001<sup>th</sup> term of the following sequence is 46, find the numerator of this term.

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$$

10. 下列加法算式中，若字母 ‘S’ 代表 4，那么字母 ‘A’ 代表甚么数字？

In the following addition, if the letter ‘S’ represents 4, what digit does the letter ‘A’ represent?

$$\begin{array}{r} \text{SEE} \\ \text{SEE} \\ \text{SEE} \\ + \text{YES} \\ \hline \text{EASY} \end{array}$$